Compensation of thermal deformation of a hybrid parallel kinematic machine

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Abstract

Many different Parallel kinematic machine architectures have been proposed, but few have actually been used in production environments. The class of machine structures referred to as hybrid parallel kinematic machines have been more successful than most and have been employed in a wide variety of applications. These machines are however still affected by many of the limitations associated with purely parallel structures. One of these is temperature-related error due to thermal expansion of the parallel machine links. This paper proposes and demonstrates a method of compensating for this error. The paper presents details of the method, developed, the development of the required kinematic model and its test and validation using a real production machine, the NEOS Tricept.

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1. Introduction

Hybrid parallel kinematic machines (HPKM)s have migrated from the research laboratory into the factory and are being utilised in a number of manufacturing installations including automotive and aerospace applications [1,2]. These applications have demonstrated their capabilities and potential for manufacturing systems based on the HPKM architecture. There are, however, a number of factors inherent in the structure of HPKM systems which limit the application of the current generation of HPKMs in high-precision applications. The error sources present in HPKMs, along with serial robots and machine tools, are multiple and interdependent [3–9] and compromise the performance of the system under real operating conditions.

One of the error sources which causes significant performance variation is the effect of thermal changes on the structure of the machine. This has been described as ‘the most significant element of the total error of long duration behaviour’ in relation to robots, of which the author considers HPKMs a subset. Thermal changes in the structure of the machine lead to geometrical variations, which cause the tool centre point (TCP) to deviate from the required position through changes to the kinematic behaviour.

By combining a generic HPKM kinematic modelling ‘toolbox’ with a real-time thermal monitoring system, it has been shown that the performance of a typical HPKM system can be improved, when compared to the uncompensated system. This system can be applied to other HPKM-structured machines with minimal empirical work. A block diagram of the system is shown in Fig. 1. It consists of several Mathlab modules which are able to calculate the effect of thermal expansion on the final position of the TCP and compensate accordingly. The target TCP position is converted to the required leg lengths using an inverse kinematics module. A correction factor is applied to these according to the readings from thermocouples and the actual position of the TCP calculated using the forward kinematic module. This is then used to calculate a compensated target position that takes into account the thermal expansion of the legs. The development of both the kinematic model and thermal compensation system are described in Section 2 and the full system implementation on a real HPKM is described in Section 3.

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2. Hybrid parallel kinematic machines

HPKMs combine parallel machine structures with more conventional serial elements. This allows the relative merits of serial systems to be combined with those of PKMs, most notably the combination of high dexterity and large work volume (serial system) with an increased level of accuracy and high degree of stiffness (parallel).

The joint and axis notations of a typical HPKM, the TR600 from Neos Robotics, are shown in Fig. 2. This machine has a parallel structure consisting of three links that provide movement in the X, Y and Z axes and a serial wrist providing three rotational axes.

2.1. Kinematic modelling

There have been a number of previous studies on the kinematic models of HPKMs with three-axis wrists [10,11], HPKMs with two-axis wrists [12] or for the parallel part of the structure only.

To develop the thermal compensation system it was necessary to be able to calculate accurately the effect of thermal expansion of the machine links on the final TCP position. This required the development of a full kinematic model. To provide this, a “HPKM kinematics toolbox” was developed using Matlab. This could then be used to perform forward and inverse kinematic transformations on individual sections or all of a mechanism having a general HPKM structure, and offered different methods for the solution of kinematic problems. Options were available for solving the inverse and forward kinematics of the tripod structure only, the wrist only or the entire structure [13].

To ensure the generic nature of the software, prior to any calculations, data are requested which describe the physical arrangement of the structure, including offsets for the wrist and the radii of the upper and lower platforms. A generic HPKM arrangement is assumed. A brief description of the techniques used for each kinematic solution is presented below.

Fig. 1. Block diagram of thermal compensation system.

Fig. 2. Tricept TR600 axis notations and base coordinate system.
2.2. Inverse Kinematics

**Tripod Structure**—The solution of the inverse kinematic problem for the tripod structure only utilises the geometric data on the structure and matrix-based calculations to calculate the pose of \( TCP_{\text{tripod}} \), a virtual point on the structure which is the interface point between the tripod and the wrist.

**Wrist**—A standard serial wrist solution utilising Denavit–Hartenberg (D–H) parameters is utilised in combination with the offset data defined for the particular structure under investigation.

**Full structure**—A simplification is utilised which treats the upper parallel structure as a single link with two angles of rotation around the upper attachment point, and a single prismatic link along its length. This allows D–H parameters to be utilised to calculate the inverse kinematic solution for the entire structure. The intermediate point of the solution, i.e. \( TCP_{\text{tripod}} \), is then passed to the tripod inverse kinematics module for solution into the 3 leg lengths.

2.3. Forward Kinematics

**Tripod structure**—The forward kinematic solution for the tripod structure uses an iterative technique based on the Newton–Raphson method and a cost–modelling function. This is described in more detail in the next section.

**Wrist**—There are two methods available for the solution of this, successive transformations and D–H parameters. The successive transformations and rotations approach utilises the values of the three joints combined with the previously inputted data on the structure of the system. The D–H parameters utilised for the inverse kinematic solution of the wrist are reused.

**Full structure**—Again, two approaches are available: the tripod forward kinematics are calculated using the iterative approach outlined above to generate the pose at \( TCP_{\text{tripod}} \), followed by successive transformations along the wrist structure to produce the pose at the system TCP; the simplified D–H parameters developed for the inverse kinematic solution are combined with those of the wrist to produce a full D–H solution for the entire structure.

2.3.1. Forward kinematic solution for hpkm parallel axes detail

A previous study [14] compared the use of this approach for PKMs against an algebraic solution based on dialytic elimination and concluded that the numerical approach was fast but required a good initial guess and can only find one solution. These disadvantages are negated in this application as the starting guess will always be close to the real solution, i.e. in the same workspace, and only one solution is possible and hence sought.

This iterative approach has been utilised for the solution of the forward kinematic problem of a hexapod PKM [15], but its use on the HPKM structure has two key differences:

- In generic terms, the working envelope of a HPKM is significantly larger than that of a hexapod PKM.
- As the tripod TCP orientation is unique for each position, there are three unknowns for a HPKM rather than six for a PKM.

The increase in working envelope will result in a higher number of iterations required to determine a solution, but this is offset by the reduced number of unknown terms.

The approach used by Whittingham was based on the use of an iterative technique and the Newton-Raphson method to calculate the “cost” of the resulting leg length error and to calculate the new value. This approach has been modified to suit the HPKM structure.

From Fig. 3, the kinematic equation for a vector chain in the PKM can be stated as

\[
L_i = R M_i + \text{Disp} - f_i \quad \text{for} \quad i = 1, \ldots, 3, \quad (1)
\]

where co-ordinate frame F is placed in the centre of the upper fixed yoke; co-ordinate frame M is placed on the plane constructed through the centres of the three lower leg joints, aligned with the central axis of the non-driven actuator; \( F_i \) is the upper joint centre denoted by the position vector \( f_i \), \( M_i \) is the lower joint centre denoted by the position vector \( m_i \), \( R M_i \) is the rotated joint centre which results from the lower platform movement denoted by \( R \), where \( R \) is the rotation matrix determined from the given lower platform location; \( \text{Disp} \) is the displacement which defines the required \((X, Y, Z)\) position; and \( L_i \) is the leg vector from the upper joint \( F_i \) to the lower joint \( M_i \).

For the inverse kinematic transformation, this is solved for \( L_i \) where all entities on the right-hand side of the equation are known. For the forward kinematic transformation the unknowns are the \( TCP_{\text{tripod}} \) pose, which in the context of the equation above are \( \text{Disp} \) (which is \((X, Y, Z)\) of the required pose) and \( R \). However, due to the influence of the central non-driven column in a HPKM, the pose itself is a function of \( \text{Disp} \) and can therefore be calculated. Rearranging the above equation in the form of a cost

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Fig. 3. Closed loop leg vector chain.
function $f(x)$ gives

$$\text{Eq. } M_j + \text{Disp}_j - L_j = f(x) \quad \text{for } i = 1, \ldots, 3, \quad (2)$$

where $x$ is used to represent the set of unknowns $(X, Y, Z)$.

Substituting the current values for $x$ into the above equations will result in a value for $f(x) = e$, which is the cost or error value. The correct set of values for $(X, Y, Z)$ will result in a cost value of zero.

The above equation is a system of three equations in three unknowns. The methodology adopted here is to find a solution for the unknowns $(X, Y, Z)$.

The partial derivatives can be found from Eq. (4).

The current value of the expression is given by $f(x_0)$ and the value of the differential coefficient of the expression at $x_0$ is given by $f(x_0)$. Thus, $x_1$, a better approximation, can be obtained, and this process can be repeated until a given set of accuracy conditions are met. Using the Newton–Raphson method requires the first derivative of the function:

$$f(x) = e_i.$$ 

As $f(x_0) = e$ can be stated as $\{x, y, z\}$, the first derivative of the full equation can be written as

$$\left( \begin{array}{ccc}
\frac{\delta L_1}{\delta x} & \frac{\delta L_2}{\delta x} & \frac{\delta L_3}{\delta x} \\
\frac{\delta L_1}{\delta y} & \frac{\delta L_2}{\delta y} & \frac{\delta L_3}{\delta y} \\
\frac{\delta L_1}{\delta z} & \frac{\delta L_2}{\delta z} & \frac{\delta L_3}{\delta z}
\end{array} \right) \begin{pmatrix} X_{n+1} - X_n \\
Y_{n+1} - Y_n \\
Z_{n+1} - Z_n \end{pmatrix} = \left[ \begin{array}{c}
-e_1 \\
-e_2 \\
-e_3
\end{array} \right]. \quad (8)$$

2.3.2. Newton–Raphson iteration

The Newton–Raphson method is a one-step iterative method that requires only the current iteration solution to calculate the next. This relies on the initial solution being close (in mathematical terms) to the real solution, but as the values under consideration are all part of a finite machine working volume, this is so. The Newton–Raphson method is generally represented by the expression

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$ 

The leg vector components can be calculated from the general method used is as follows:

i. input required leg lengths,
ii. define initial guess at TCP position,
iii. calculate leg lengths at starting position,
iv. use cost function to calculate leg errors and compare to error tolerance,
v. use Newton-Raphson method to generate the next guess for TCP position,
vi. repeat until errors are within tolerance,
vii. when tolerance is met, output the final TCP position and orientation.

2.3.3. Parallel axes forward kinematic model validation and testing

As stated previously, it was anticipated that the number of iterations required to find a solution with the required precision would be greater for a HPKM structure than for a hexapod PKM due to the greater potential working volume. Both the number of iterations required to reach the final solution and the trade-off between the required precision and number of iterations were tested.

2.3.4. Convergence testing

Initial testing of the model resulted in very high numbers of iterations being required due to the nature of the tripod structure of a HPKM. A small increase in leg length causes a large variation in the $(X, Y, Z)$ position of TCPtripod, thus, when the differential is calculated, as the slope of the line is shallow, a very small increment results. To overcome
this, a factor was introduced to amplify the increment by 500. This value was optimised to produce a solution in a reasonable number of iterations, while ensuring convergence over a typical working volume.

The iteration testing was completed using a number of target positions and a typical test is illustrated in Figs. 4–7.

For this case the leg lengths are $l_1 = -924.424$, $l_2 = -683.585$ and $l_3 = -725.313$ mm, which gives a target TCP_tripod position of (400 500 1300). Note that the distance between the start point and the target is 948.68 mm. It should also be noted that the recorded leg lengths are not the absolute distance between the upper and lower gimbals,
but use the manufacturer’s proprietary notation system. To ensure consistency, a conversion was performed on the true values to ensure that the results can be compared directly to those reported by the controller.

2.4. HPKM thermal errors

When investigating the effect of heat on a typical HPKM, Heisel observed temperature changes of 15–20 °C in the tripod arms where the motors are mounted and 40 °C in the wrist area. The effect of the thermal changes was a drift of 0.45 mm in the Z-axis, and 0.1 mm in the X and Y axes. The conclusion of this work was “the necessity for compensational and design measures with which the causes and effects of thermal errors can be reduced” [6].

2.5. Measurement of thermal errors

To determine the effect of thermal errors on the structure of a typical HPKM, experimental work was undertaken in which thermocouples were attached to ‘hot spots’ on the machine structure.

These positions were determined by identifying the hottest location on the outside of the structure associated with each of the joint motors. The advantage of this approach is that it is easy to retrofit the thermocouple system to an existing machine, where access to the internal structure close to the motors and key areas of thermal significance is difficult. The disadvantage of this approach is that there is a thermal lag for each measurement as the true temperature cannot be recorded.
The HPKM utilised for this work was a Neos Robotics Tricept TR600. Duty cycles designed to cause a significant amount of thermal change were programmed into the machine, both for single and multiple axis motion. Periodically, the robot was returned to a reference position where the change at the TCP was measured using a three-gauge measurement system. The measurement station comprised three Mitutoyo digital dial indicators interfaced via a multiplexer to a PC, as shown in Fig. 8. The measurement station is moveable and can be located anywhere within the working envelope of the Tricept.

Experiments were undertaken to determine the nature and magnitude of the TCP drift for heating cycles associated with both the upper tripod joints and those in the wrist. As an example, for Joint 2, in the upper tripod, the joint was extended and retracted 300 mm at full speed 25 times between each return to the measurement station. The change in gauge measurement was recorded for 100 cycles. It was observed from this work that the recorded temperature of Joint 2 increased by 15 °C and the measured TCP drift was 0.7 mm in the X-axis, with a smaller dislocation of 0.2 mm in the Z-axis. No significant dislocation was measured in the Y-axis, which is as expected due to the location of Joint 2 within the tripod. This measured TCP drift is shown in Fig. 9.

A similar test was undertaken for Joint 3 and the measured TCP drift is shown in Fig. 10. The recorded temperature increase during the test was 16 °C and the drift recorded was 0.4 mm in X, −0.65 mm in Y and 0.25 mm in Z. The symmetrical nature of the structure allows the results for Joint 3 to be applied to Joint 1 with the appropriate signs inverted.

For the joints in the robot wrist tests which moved the joint ±90° a number of times between measurements were undertaken, and for all three joints significant deflection
(>0.05 mm) was measured only in the Z-axis at the measurement station. The results for joints 4, 5 and 6 in the wrist are shown in Fig. 11.

From Fig. 12 it can be seen that the level of TCP deflection measured is smaller than that associated with the heating of the joints in the upper tripod of the machine. It is not possible to exercise the joints in the wrist in exactly the same way as those in the tripod, due to the inherent differences between them, but the motion cycles to induce the heating were designed to cause the greatest amount of heat build-up in the shortest possible time. It is considered that these types of movements would not be performed in a production environment and hence the measurements here are the worst case.

2.6. Thermal compensation

As the structure of a typical HPKM has two distinct parts, the upper tripod and the wrist, the compensation approach must also be split into two parts. For the joints in the wrist, the effect due to thermal change exhibits the same effect as TCP growth, i.e. the apparent length of a cutting tool increases. There is therefore a straightforward mechanism to compensate for this, provided that there is a good correlation between the recorded temperature and the resulting amount of TCP growth.

The upper tripod joints are more complex and the correlation between the measured joint temperature and the predicted leg length change must be considered more carefully. A complication is the fact that the motion of a single leg, as measured in the previous section, causes secondary heating in the other non-driven legs, through heat build-up due to friction. To fully determine the effect of a single leg is complex and hence a simplified approach is required. Let us assume that the general profile of the actuator is a rod with a heat source at one end. Knowing the thermal properties of the material, its length and the temperature at each end, the change in rod length can be calculated from

$$\Delta l = 0.5ax(t_1 - t_2), \quad (12)$$

where $l$ is the rod length and $x$ the coefficient of thermal expansion.

The key limitations of this approach are that the recorded temperature will not be the true rod-end temperature, the rod itself is partially shrouded, and the nature of the shrouding changes as the rod length increases and the effect of secondary heating at the nut is not included.

To overcome this, a ‘CTE factor’ is included in the above equation, which allows the measured leg length changes to be matched to those obtained from the equation. A dataset obtained from a full structure thermal test was used as

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**Fig. 11.** Measured TCP drift during wrist joint heating.

**Fig. 12.** Predicted vs. actual error vector—Joint 2.

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the input to the thermal model for the full structure, and the predicted TCP deviations were compared to the true values. The two sets of data exhibited the greatest level of correlation when the CTE factor was 4. This value can now be used to compare the predicted error vector magnitude to the measured error vector for a single actuator in the upper tripod. The results are shown in Fig. 14. It can be seen that the prediction method underestimates the level of TCP deflection, but that the level of underestimation decreases as the heating cycle length increases. The underestimation is due to the uneven heating of the strut while the model is based on a constant thermal state between the motor casing and the lower part of the strut.

For each of the joints in the wrist, an equation was derived which related the recorded temperature to the change in apparent TCP length measured. This was performed iteratively to isolate the heating effects for each joint from the others. This is due to the close proximity of Joints 5 and 6 in the wrist structure.

3. Implementation

The final implementation of the thermal compensation methodology is shown schematically in Fig. 13. The general principle is that, prior to a critical positional move, the current temperature values are read and passed to the Labview application containing the forward kinematic model developed in Section 2 which calculates the predicted deflections that will arise. These are split into two, those that affect the apparent tool length and those that affect the entire structure. The predicted deflections are inverted and used as the compensation values that are applied. The tool length changes are written to the Tricept controller as a text file containing a new TCP offset, and the overall structure changes are written as a text file containing a new base offset.

4. Validation

A simulation-based validation was initially undertaken to compare the results from the Labview module to those obtained empirically. A dataset of thermocouple readings was input into the model and the predicted deviation compared with the measured one. The results of this are shown in Fig. 14 and it can be seen that a significant proportion of the error is predicted, and that the level of correlation increases as the number of cycles increases.

A number of empirical validation tests were also undertaken, which included single and multiple joint tests. This culminated in an all-axis validation test in which the compensation values were applied immediately prior to return to the measurement station. The level of TCP deviation was recorded over a number of motion cycles and compared to the original values obtained during the data-gathering. The results are shown in Fig. 15.

The results of this show that the error at the TCP resulting from thermal drift has been significantly reduced. For the duty cycle undertaken here, the resulting error after 10 cycles without temperature compensation is 0.2 mm, but with the compensation system active this is reduced to 0.03 mm, a reduction of 84%.

Fig. 13. Schematic of temperature compensation system.

Fig. 14. Error vectors—predicted vs. simulated.
5. Conclusions

A generic HPKM Kinematics Toolbox has been developed, which permits forward and inverse kinematic solutions to be calculated using a number of techniques for part or all of a generic HPKM structured system. This shows good correlation to a real system when calibration effects are taken into consideration.

The implementation of this system has allowed a real-time thermal error compensation system to be developed, which allows thermal changes to the structure of a machine to be compensated for at the tool tip prior to any critical machine movements. The hardware requirements of the thermal modelling system are minimal and different systems can be monitored due to its generic nature.

In combination, it has been shown using a typical HPKM that the level of error exhibited by the machine under thermally unstable conditions can be controlled to a high degree and that the thermal compensation system can ensure that the system performance remains within the levels required for higher accuracy applications such as those found in the aerospace industry.

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